PECULIARITIES OF THE FLOW OF A VIBRATIONALLY RELAXING GAS IN A NOZZLE WITH A SEGMENT OF CONSTANT CROSS SECTION IN THE THROAT REGION

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1. A large number of reports have been devoted to a numerical investigation of nonequilibrium flows in nozzles. A detailed bibliography on this field can be found in [1-3]. In addition, we point out the recently published reports [4-11] on the calculation of two-dimensional flows of a gas mixture in nozzles with relaxation of vibrational energy. At the same time, it should be mentioned that among the large number of publications devoted to this question one can name a relatively small circle of reports [6-11] in which sub-, trans-, and supersonic flows of a vibrationally relaxing gas in the vicinity of the minimum cross sections of narrowing–expanding nozzles were investigated. However, it is just here, in the region of large negative temperature and pressure gradients, during the passage of the stream through the speed of sound, that the physicochemical conversions occur which largely determine the character of the processes in the supersonic part of a nozzle.

We note that flows in nozzles whose minimum cross sections were formed by smoothly joined arcs of circles or smooth curves were considered in [6-10]. It was shown that separation of the vibrational temperatures from the translational temperature of the gas mixture sets in the transonic region of flow before the critical cross section and develops downstream in the supersonic part of the nozzle.

Nozzles with a critical cross-section height (diameter) $h_* = 0.4$-1.0 mm and having a segment of constant height in the throat region, as a rule, have recently found wide application, however. A positive longitudinal pressure gradient at the wall in the throat was noted in [11] in a calculation of the flow of a vibrationally relaxing gas in such a nozzle with a segment of constant cross section in the throat of length $l_c = 0.6$ [$l_c = L_c/(h_*/2)$ and L_c is the length of the segment of constant cross section]. The results of an experimental investigation of the gasdynamic structure of the stream in the throat region of a supersonic nozzle with $l_c = 1.0$ are presented in [12]. Later (in a continuation of [12]) R. K. Tagirov made calculations of the flows of an inviscid and thermally nonconducting ideal gas in narrowing–expanding nozzles with $l_c = 0.4$-2.0 by S. K. Godunov's method of first-order accuracy, which showed satisfactory agreement with experiment [12].

However, for a careful analysis of the peculiarities of flow in regions of small extent (especially in the vicinity of the minimum cross section of a Laval nozzle) it is advisable to use more precise methods of determining the gasdynamic and thermodynamic structure of the stream. Moreover, the use of difference schemes of an increased order of accuracy allows one to obtain reliable results on coarser grids than when schemes of a lower order of approximation are used. In this case one can achieve a significant reduction in the expenditures of machine time required to calculate each variant.

In this report we present the results of a numerical investigation of the steady, two-dimensional, mixed sub- and supersonic flow of an inviscid and thermally nonconducting, vibrationally relaxing gas in plane narrowing–expanding nozzles having a segment of constant cross section in the throat region. The results are obtained in a joint solution of the equations of gas dynamics and relaxation kinetics on the basis of the establishment method developed in [13] for solving the direct problem of gas flow in a Laval nozzle in the presence of nonequilibrium physicochemical processes, using a high-precision difference scheme [14].

2. We consider mixed, sub- and supersonic, plane or axisymmetric, nonequilibrium flow in a narrowing–expanding nozzle of a relaxing gas mixture having a stagnation temperature and pressure T_0 and p_0 in the reservoir. It is assumed that irreversible physicochemical

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processes, which are characterized by the variation of N dimensionless parameters q_n, n = 1, 2, ..., N, take place in the medium. Together with the pressure p and the density ρ, these parameters determine the nonequilibrium state of the medium.

The complete system of nonsteady, two-dimensional, divergent equations describing the plane (ν = 0) or axisymmetric (ν = 1) flow of an inviscid and thermally nonconducting gas in the presence of nonequilibrium physicochemical processes is written in the vector form

$$\frac{\partial \mathbf{f}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{g}, \tag{2.1}$$

where f, F, G, and g are column vectors:

$$\mathbf{f} = \rho \begin{pmatrix} 1 \\ u \\ v \\ 2e + w^2 \\ q_n \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \rho u \\ p + \rho u^2 \\ \rho uv \\ \rho u (2i + w^2) \\ \rho u q_n \end{pmatrix},$$
$$\mathbf{G} = \begin{pmatrix} \rho v \\ \rho uv \\ p + \rho v^2 \\ \rho v (2i + w^2) \\ \rho v q_n \end{pmatrix}, \quad \mathbf{g} = -\begin{pmatrix} \nu \rho v / y \\ \nu \rho v u / y \\ \nu \rho v^2 / y \\ \nu \rho v (2i + w^2)/y \\ \nu \rho v q_n / y - \rho \Phi_n \end{pmatrix}. \tag{2.2}$$

Here ρ is the density; u and v are the projections of the vector of velocity w onto the x and y axes of the Cartesian coordinate system; p is the pressure; e is the specific internal energy; i is the specific enthalpy; Φ_n are known functions of p, ρ, and q. The system of differential equations (2.1), (2.2) under consideration is closed by the caloric equation of state

$$i = i(p, \rho, \mathbf{q}). \tag{2.3}$$

The unknown functions at the nozzle entrance are overdetermined with allowance for the assigned values of the total enthalpy and entropy, it being assumed that $v \equiv 0$ and $\partial u/\partial x \equiv 0$. The boundary conditions at the nozzle walls are assigned from the condition of nonpenetration. The values of the unknown functions at the exit, in the supersonic part of the nozzle, are found using linear extrapolation.

All the required information about the method of numerical solution of the direct problem of nonequilibrium gas flow in a Laval nozzle being used is contained in [13, 14].

We additionally emphasize the fact that the equations of nonequilibrium kinetics belong to the type of so-called "hard" equations or equations with a small parameter in the leading derivative. The solution of equations of this type by the methods widely popular in gas dynamics does not allow one to carry out integration with a time step τ considerably exceeding the characteristic time of occurrence of the physicochemical processes (the vibrational relaxation time). This results in considerable expenditures of machine time.

To increase the permissible step τ of numerical integration in the present work we used a method proposed in [15, 16] and consisting in the expansion of the source terms Φ_n in series in q_n up to first-order derivatives, inclusively. This method allowed us to increase the step τ to the value determined from the condition of stability of the difference scheme [14], and it considerably reduces the required expenditures of machine time. To reduce the time of calculation of each variant, the initial distributions of the parameters along the nozzle were assigned from the solution of the problem under consideration in the one-dimensional steady-state approximation.

3. The results of calculations of the steady-state, two-dimensional, mixed flow of the vibrationally relaxing gas mixture 10% CO_2 + 88% N_2 + 2% H_2O in plane narrowing—expanding nozzles having a segment of constant height in the throat region and two points of a break in the contour at the start and end of the segment of constant cross section are presented below. In the calculations the corner points are replaced by arcs of circles of sufficiently small radius r = 0.3. All the linear dimensions are given in dimensionless form. The half-height

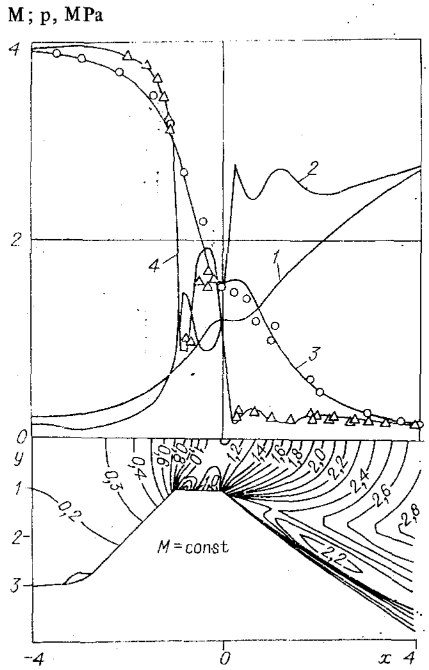

Fig. 1

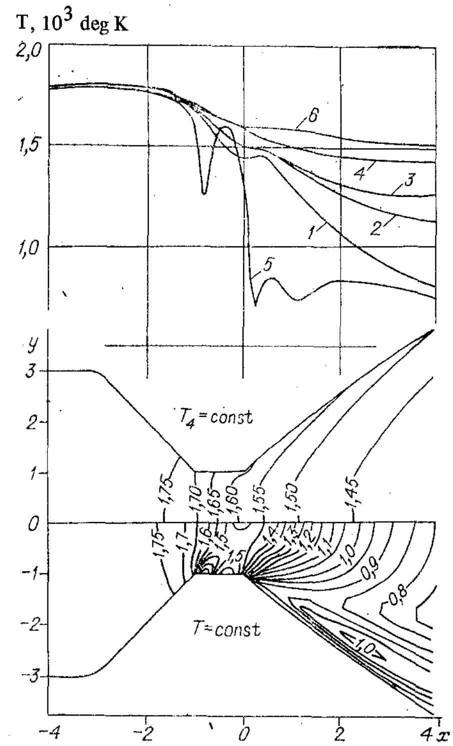

Fig. 2

$h_*/2$ of the critical cross section is chosen as the characteristic linear scale. The nozzles have critical cross sections of the same height $h_* = 0.4$ mm but differ from each other in the length of the segment of constant cross section in the throat region: $l_c = 1, 2, 4$. The subsonic parts of the nozzles have a wedge-shaped entrance with a total aperture angle of 90° and the supersonic part is profiled to an expansion ratio of 28. The stagnation temperature and pressure of the mixture in the nozzle reservoir are taken as $T_0 = 1800$°K and $p_0 = 4$ MPa.

As the nonequilibrium parameters characterizing the vibrational state of the medium we take the average number of vibrational quanta per type of vibrations of a molecule (without degeneration) $q_n = [\exp(\theta_n/T_n) - 1]^{-1}$ (θ_n and T_n are the characteristic and the nonequilibrium vibrational temperatures of the n-th type of vibrations, n = 1, 2, 3, 4), representing the dimensionless expressions for the vibrational energies of the corresponding types of vibrations, with q_1, q_2, and q_3 corresponding to the three respective types of vibrations of the CO_2 molecule and q_4 to vibrations of the N_2 molecule. The equations of vibrational relaxation in the system (2.1), (2.2), and the caloric equation of state (2.3) are written in the form given in [2], from which we also take the times of V—T exchange and the constants of V—V and V—V' exchange.

In Figs. 1 and 2 we present the results of a calculation of the flow of the vibrationally relaxing gas mixture under consideration in a plane narrowing—expanding nozzle having a segment of constant height with a length $l_c = 1$ in the vicinity of the critical cross section. The field of Mach number in isolines of M = const is shown in Fig. 1. The variation of the Mach number along the axis and the wall is shown by curves 1 and 2, respectively. The variation of the pressure p along the axis and the wall (curves 3 and 4, respectively) is compared with experiment [12] there also, with allowance for the fact that in the region of the minimum nozzle cross section the departure of the three-component gas mixture under consideration from equilibrium is slight and has little effect on the gas dynamics of the stream at equal adiabatic indices.

The fields of the translational temperature T of the mixture (lower half-plane of the nozzle) and the vibrational temperature T_4 of nitrogen (upper half-plane) are shown in Fig. 2 in isolines of T_n = const; the variation along the axis of the translational temperature T (curve 1), the vibrational temperatures of the doubly degenerate, symmetric deformation (T_2) and antisymmetric valence (T_3) types of vibrations of the CO_2 molecule (curves 2 and 3), and

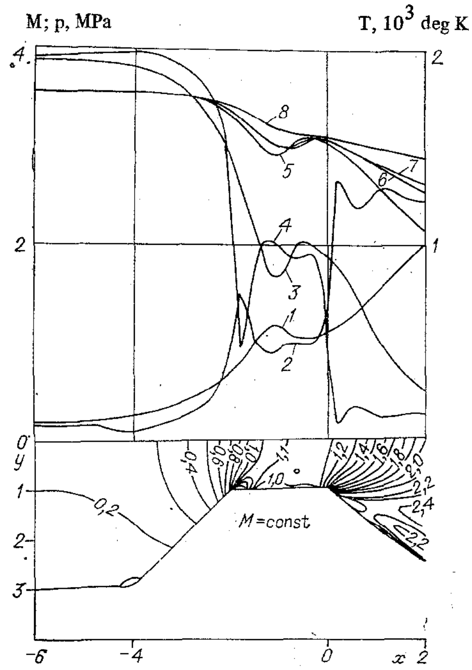

Fig. 3

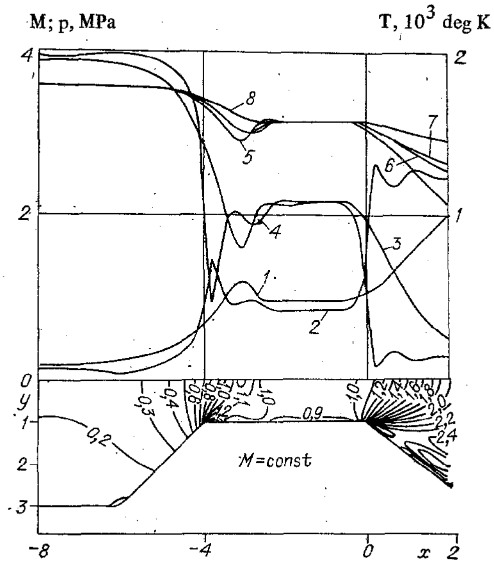

Fig. 4

the vibrational temperature T_4 of nitrogen (curve 4) is shown. The variation of the translational (T) and vibrational (T_4) temperatures at the wall is shown by curves 5 and 6, respectively.

An analysis of the results presented in Figs. 1 and 2 shows that flow over the corner points located at the start of the segment of constant cross section is accompanied by overexpansion of the stream to supersonic velocity with subsequent deceleration in the compression shocks. For l_c = 1 the shock developing behind the corner point at the start of the segment of constant cross section reaches the nozzle axis in the vicinity of the exit from the throat and weakens here, being blurred by the rarefaction wave formed in flow over the corner plot located at the end of the segment of constant cross section. Separation of the vibrational temperature of nitrogen from the vibrational temperatures of CO_2 and of the latter from the translational temperature of the mixture takes place in this section of supersonic overexpansion of the stream, i.e., the process of freezing in of the energy of the vibrational degrees of freedom in the nitrogen and CO_2 molecules takes place. The subsequent passage of the stream through the compression shocks, however, is accompanied by an increase in the translational temperature of the mixture and by the intensive process of relaxation of vibrational energy in the nozzle throat. The mixture tends toward equilibrium in the translational–rotational and vibrational degrees of freedom, but does not reach it. In this case the flow behind the compression shock remains supersonic and a small zone of subsonic flow forms only near the wall. Further separation of the vibrational temperatures of nitrogen and CO_2 from the translational temperature of the mixture and among themselves occurs in the supersonic part of the nozzle.

With an increase in the length of the segment of constant cross section the flow pattern changes. The field of Mach numbers for a nozzle with l_c = 2 is represented in isolines of M = const in Fig. 3, analogous to Fig. 1, and the variation of the Mach number and the pressure p at the axis and the wall is shown in curves 1-4. In addition, the variation of the translational temperature T and the vibrational temperatures T_2, T_3, and T_4 along the axis is shown by curves 5-8, respectively.

It is seen that for l_c = 2 the compression shock formed behind the corner point located at the start of the segment of constant cross section reaches the opposite wall at the exit from the throat. In this case the intensity of the compression shock and the losses of vibrational energy in the throat grow somewhat and the zone of subsonic flow near the wall increases. In the core of the stream the flow remains supersonic as before, however, and the separation of the vibrational temperature of nitrogen from the translational temperature of the mixture and the vibrational temperatures of CO_2 is retained.

With a further increase in l_c the flow pattern changes qualitatively. The results of a calculation of the flow of the vibrationally relaxing gas mixture under consideration in a nozzle with $l_c = 4$ are presented in Fig. 4. The field of Mach numbers, the variation of M and p at the axis and the wall, and the variation of T, T_2, T_3, and T_4 at the axis are shown by analogy with Fig. 3. We note that for $l_c = 4$ the flow behind the compression shocks becomes subsonic and passes through the speed of sound again in the flow over the corner points located at the end of the segment of constant cross section. In this case the separation of the vibrational temperatures from the translational temperature of the gas mixture which formed in the flow over the corner points at the start of the throat does not develop downstream, as occurs in nozzles in which the vicinity of the critical cross section is formed by smoothly joined curves but disappears as the stream passes through the compression shocks and slows in the throat. The system reaches equilibrium, but now at a lower temperature level, and departs from it again in the supersonic part of the nozzle.

Thus, the complete loss of the vibrational energy stored in the subsonic part of the nozzle occurs for $l_c > 2$.

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